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An Information-aware Lyapunov-based MPC for autonomous robots

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***Abstract*— This paper proposes a feedback-feedforward control scheme that combines the benefits of an online active sensing control strategy to maximize the information needed for correctly executing the desired task (the feedforward component), with a Lyapunov-based control strategy that guarantees an asymptotic convergence towards the task itself (the feedback component). To show the effectiveness of our method, we consider a unicycle equipped with onboard sensors that has to perform the classical path following task.**

I. INTRODUCTION

In robotics, action, and motion planning [1] are typically used to accomplish a given task (e.g., reaching a particular configuration) with stability guarantees (e.g., Lyapunov stability theory), and/or optimizing a cost of interest (e.g., control effort), under different constraints (e.g., on limited Field-of-View sensors). However, as for humans, the successful generation and execution of a motion plan substantially depends on the accuracy of the reconstructed surroundings and (internal) state trajectories that, in a real scenario, are not assumed directly measurable by on-board sensors but only estimated by using a nonlinear filter whose performance depends on the acquired sensory information. Due to nonlinearities, the quality of the sensory information strongly depends, as for humans, on the actions chosen to perform the task. This paper proposes a feedback-feedforward strategy for a robotic system where the feedforward component aims at maximizing the information collected through the onboard sensors by using an online active sensing control strategy [2], while the feedback component guarantees an asymptotic stability, in the Lyapunov sense, of the desired task. To quantify the amount of the collected information along the planned trajectories, the *Constructability Gramian* (CG) is used as the guiding metric, while the effective combination of the feedback/feedforward components is pursued by adopting a Lyapunov-based Model Predictive Control (LMPC). For the sake of space, this work shows only some simulation results obtained by applying our method. Please refer to the extended version [3] and to the accompanying multimedia material for further details and simulation results.

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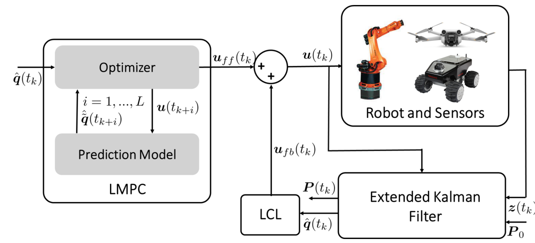

Fig. 1. Feedback-feedforward control scheme that determines at runtime the optimal feedforward control action that maximizes the information collected through sensors along the future path taking into account the feedback control action that guarantees the asymptotic convergence toward the desired task.

II. PROPOSED METHODOLOGY

The components of the proposed feedback-feedforward control scheme Fig. 1 are detailed in this section.

A. THE FEEDBACK COMPONENT

Let us consider a time-invariant, input affine nonlinear system $\dot{\boldsymbol{q}}(t) = \boldsymbol{f}(\boldsymbol{q}(t)) + \boldsymbol{g}(\boldsymbol{q}(t))\boldsymbol{u}(t)$ where $\boldsymbol{q}(t) \in \mathbb{R}^n$ is the state of the system, $\boldsymbol{u}(t) \in \mathbb{R}^m$ its control input and $\boldsymbol{f}(\cdot)$ and $\boldsymbol{g}(\cdot)$ are the drift vector and the control vector field, respectively. Let us then consider a positive definite candidate of Lyapunov $V(\boldsymbol{q}(t))$, with $V(0) = 0$ and $\boldsymbol{q}(t) = 0$ the desired equilibrium. The Lyapunov-based Control Law $\boldsymbol{u}(t) = \boldsymbol{u}_{fb}(\boldsymbol{q}(t))$ (LCL in Fig. 1) that makes $\boldsymbol{q}(t) = 0$ asymptotically stable is derived by imposing $\dot{V}(\boldsymbol{q}(t)) = L_f V(\boldsymbol{q}(t)) + L_g V(\boldsymbol{q}(t))\boldsymbol{u}_{fb}(\boldsymbol{q}(t))) \leq 0$. The above control design implicitly assumes that the state of the nonlinear system $\boldsymbol{q}(t)$ is completely known. However, in a real scenario, the state of the system is usually unknown, and only an estimate $\hat{\boldsymbol{q}}(t)$ is made available by an observer which exploits sensory data. As a consequence, the control inputs $\hat{\boldsymbol{u}}_{fb}(\hat{\boldsymbol{q}}(t))$ are computed on the state estimates, which of course are affected by uncertainties. For this reason, the time derivative of V becomes

$$\dot{V}(\boldsymbol{q}(t), \hat{\boldsymbol{q}}(t)) = L_f V(\boldsymbol{q}(t)) + L_g V(\boldsymbol{q}(t))\hat{\boldsymbol{u}}_{fb}(\hat{\boldsymbol{q}}(t)). \quad (1)$$

Since an EKF will be adopted as an observer for the state estimation, we can assume that up to the first order $\hat{\boldsymbol{q}} = \boldsymbol{q} + \boldsymbol{\varepsilon}_q$, with $\mathrm{E}\{\boldsymbol{\varepsilon}_q\} = 0$ and, assuming that $\boldsymbol{\varepsilon}_q$ is the estimation error and $\boldsymbol{P}$ its covariance matrix returned by the EKF, we have $\boldsymbol{P} = \mathrm{E}\left\{\boldsymbol{\varepsilon}_q \boldsymbol{\varepsilon}_q^T\right\}$. It then follows that the two moments of (1) depend on the state estimation uncertainty that, hence, also affect the stability of the equilibrium.

B. THE FEEDFORWARD COMPONENT

Let us consider now the classical Lyapunov-based MPC described in [4, Chapter 2]. With respect to it we have to deal with two major issues. First, only an estimate of the state $\hat{\boldsymbol{q}}(t_k)$ is available at time t_k, implying that $\boldsymbol{u}(t_k)$ turns to $\hat{\boldsymbol{u}}(t_k)$. Second, the proposed feedback-feedforward control scheme assumes that the first step of the MPC synthesized control law fed to the system is given by $\hat{\boldsymbol{u}}(t_k) = \hat{\boldsymbol{u}}_{ff}(t_k) + \hat{\boldsymbol{u}}_{fb}(t_k)$ (see Fig. 1). Moreover, in this active sensing setting, the cost function introduced in [5] is adopted. As a consequence, the Information-aware LMPC problem to be solved at runtime for the feedback-feedforward control scheme reads as follows:

Problem 1 (Information-aware LMPC) *Given the prediction horizon L, the control input $\hat{\mathbf{u}}(t)$, the predicted trajectory of the estimated system $\hat{\hat{\boldsymbol{q}}}$ and with initial state $\hat{\boldsymbol{q}}(t_k)$ at time t_k, find, $\forall t \in [t_k, t_{k+L}]$, the optimal feedforward control components*

$$\boldsymbol{u}_{ff}^* = \min_{\boldsymbol{u}_{ff} \in S(\Delta)} -\|\boldsymbol{\mathcal{G}}_c(-\infty, t_{k+L})\|_\mu \tag{2}$$

s.t.

$$1)\ \dot{\hat{\hat{\boldsymbol{q}}}}(t) = \boldsymbol{f}(\hat{\hat{\boldsymbol{q}}}(t)) + \boldsymbol{g}(\hat{\hat{\boldsymbol{q}}}(t))(\boldsymbol{u}_{ff}(t) + \boldsymbol{u}_{fb}(\hat{\hat{\boldsymbol{q}}}(t))) \tag{3}$$

$$2)\ \hat{\hat{\boldsymbol{q}}}(t_k) = \hat{\boldsymbol{q}}(t_k) \tag{4}$$

$$3)\ \underline{\boldsymbol{u}} - \boldsymbol{u}_{fb}(\hat{\hat{\boldsymbol{q}}}(t)) \le \boldsymbol{u}_{ff}(t) \le \bar{\boldsymbol{u}} - \boldsymbol{u}_{fb}(\hat{\hat{\boldsymbol{q}}}(t)) \tag{5}$$

$$4)\ L_g V(\hat{\boldsymbol{q}}(t_k))\hat{\boldsymbol{u}}_{ff}(t_k) \le 0 \tag{6}$$

where $S(\Delta)$ is the family of piece-wise constant functions with sampling period Δ, (3) *is the model of the system, which is used to predict the state evolution starting from the initial estimated state* (4)*,* (5) *are the control constraints and* (6) *is the stability constraint.*

III. SIMULATION RESULTS

To prove the effectiveness of our approach, we test it on a unicycle vehicle that have to perform a path following task. We compare the results applying the proposed Information-aware LMPC (dubbed I-LMPC), i.e., feedback-feedforward controls obtained by the solutions of Problem 1, with: 1) the results obtained by directly applying the feedback $\hat{\boldsymbol{u}}_{fb}(t)$ only (LCL) 2) the results obtained by applying the solution of Problem 2 where the cost function is a task-oriented cost function (classical LMPC). We perform 100 simulations and we carry out a statistical analysis in terms of estimation error and task error by using a Wilcoxon rank sum test with a significance level of 5%. All the optimization problems are solved using the *CasADi* tool in Python and adopting the direct single shooting method with the *ma57 ipopt solver*.

A. Path following

The objective is to determine a Lyapunov based control law such that the vehicle is asymptotically stabilized, w.l.o.g. on the straight line $y = 0$. Note that for this task, the dynamic of y and θ are not influenced by the one of x. In the simulations, the unicycle is equipped with a sensor that measures the range from the path, i.e., $h(t) = y(t)$. Notice that, the straight line $y = 0$ is an unobservable path with this output (θ is not observable). We choose $\Delta = 0.05$ s, $L = 30$, $T = 18$ s and the initial configuration is $q_0 = [5\text{ m},\ \pi\text{ rad}]^T$ with $\boldsymbol{P}_0 = \text{diag}([0.5^2,\ 0.2^2])$. Moreover, we assume a normally distributed Gaussian output noise with zero mean and covariance matrix, $\boldsymbol{R} = 0.3\boldsymbol{I}$ while the actuation/process noise is considered negligible. To conclude, $\bar{v} = 1$ m/s and $-7 \le \omega_{fb} + \omega_{ff} \le 7$. Fig. 2 shows the mean values with their standard deviation of both the estimation errors and the task execution performances and of the smallest eigenvalue of P^{-1}. For the LCL and the LMPC, the RMS of estimation errors do not converge to zero and hence the task is not correctly executed. In addition, for the I-LMPC case, the uncertainty is smaller than the other two cases most of the time. Notice that, as soon as the vehicle approaches the desired path the smallest eigenvalue of $\boldsymbol{P}^{-1}$ reduces, confirming that the straight line $y = 0$ is an unobservable path. Moreover, the Wilcoxon test confirms that there are statistical differences for all cases, and that our approach provides the most informative trajectories.

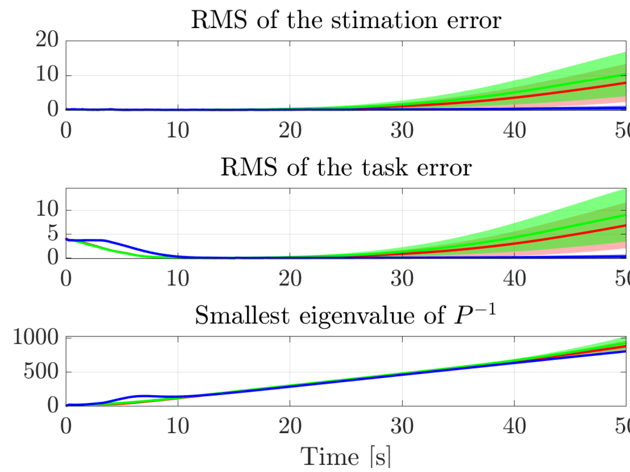

Fig. 2. Statistical results in terms of average value and standard deviations for the path following task. The results obtained for the LCL are plotted in red, for the LMPC in green, for the proposed I-LMPC in blue.

IV. CONCLUSIONS AND FUTURE WORKS

This paper proposed a feedback-feedforward Information-aware LMPC control scheme that combines the benefits of an online active sensing control strategy and a Lyapunov-based control strategy. Future works will deal with the extension of our methodology to a risk-aware control scheme where the feedforward component maximize the information on the surrounding risks while the feedback component is used for the task execution in a risky environment.

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